

Electromagnetic form factors of nucleons and $p \rightarrow \Delta^*$

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Abstract

A relativistic quark model and a new set of wave functions of nucleon and Δ have been used to study the electromagnetic properties of $\frac{1}{2}^+$ baryons and photoelectric production of $\Delta(1236)$. Theoretical results of $G_M^p(q^2)$, μ_p , μ_Λ , $\mu_p G_E^p(q^2)/G_M^p(q^2)$, $G_E^n(q^2)$, and $G_{M1+}(q^2)$, μ , E_{1+} , S_{1+} of $p \rightarrow \Delta$ are presented.

1 Introduction

The relativistic quark model [1] is successful in studying the electromagnetic and weak interactions of ground-state baryons and mesons. However, some results are inconsistent with experimental data[3], for instance, theoretical ratio $\mu_p G_E^p(q^2)/G_M^p(q^2)$ drops faster with q^2 than experimental values. On the other hand, recently there has been some experimental results about the electromagnetic properties of ground-state baryons, for example, the measurement [4,5] of the magnetic transition form factor $G_{M1+}(q^2)$ for $p \rightarrow \Delta^+(1236)$. These results need theoretical explanation. In Ref.[2], a new set of baryon wave functions have been constructed by requiring $SU(6)$ symmetry in the frame of center of mass. For example, in the wave function of $\frac{1}{2}^+$ baryon there are additional terms

$$\{(1 + \frac{i}{m}\hat{p})\gamma_5 C\}_{\alpha\beta} u_\lambda(p)_\gamma + 2C_{\alpha\beta}\{\gamma_5 u_\lambda(p)\}_\gamma. \quad (1)$$

p is the momentum of the baryon, m is the mass, and $C = i\gamma_2\gamma_4$ which is the charge-conjugation operator. The new wavefunctions are still s-wave and satisfy $SU(6)$ symmetry in the frame of center of mass. It is interesting to point out that the original wave function

$$\{(1 - \frac{i}{m}\hat{p})\gamma_5 C\}_{\alpha\beta} u_\lambda(p)_\gamma$$

is constructed by the spinors of quarks with zero momentum and the new terms(1) are constructed by the spinors of antiquarks with zero momentum. Both satisfy $SU(6)$ in the frame of center of mass.

In this paper, the method proposed in Ref.[1] and the wave functions constructed in Ref. [2] are used to study the electromagnetic properties of nucleons and $p \rightarrow \Delta^+(1236)$.

2 Matrix element of electric currents

The effective Hamiltonian of electromagnetic interaction in quark model [1,2] is

$$H_i(x) = -ie\bar{\psi}(x)Q\{\hat{A}(x) - \frac{i\kappa}{4m_p}\sigma_{\mu\nu}F_{\mu\nu}(x)\}\psi(x). \quad (2)$$

The wave function of $\frac{1}{2}^+$ baryon is[2]

$$\begin{aligned} B_{\alpha\beta\gamma,ijk}^{\frac{1}{2}\lambda}(x_1, x_2, x_3)_l^{l'} &= \frac{1}{6\sqrt{2}}\sqrt{\frac{m}{E}}\varepsilon_{i'j'k'}\{\Gamma_{\alpha\beta,\gamma}(p)_\lambda(\varepsilon_{ijl'}\delta_{kl} + \varepsilon_{ikl'}\delta_{jl}) \\ &\quad + \Gamma_{\beta\gamma,\alpha}(p)_\lambda(\varepsilon_{jkl'}\delta_{il} + \varepsilon_{ikl'}\delta_{jl})\}, \\ \bar{B}_{\alpha\beta\gamma,ijk}^{\frac{1}{2}\lambda}(x_1, x_2, x_3)_l^{l'} &= -\frac{1}{6\sqrt{2}}\sqrt{\frac{m}{E}}\varepsilon_{i'j'k'}\{\bar{\Gamma}_{\alpha\beta,\gamma}(p)_\lambda(\varepsilon_{ijl'}\delta_{kl} + \varepsilon_{ikl'}\delta_{jl}) \\ &\quad + \bar{\Gamma}_{\beta\gamma,\alpha}(p)_\lambda(\varepsilon_{jkl'}\delta_{il} + \varepsilon_{ikl'}\delta_{jl})\}. \end{aligned} \quad (3)$$

$$\begin{aligned} \Gamma_{\alpha\beta,\gamma}(p)_\lambda &= \{[f_1(x_1, x_2, x_3) - \frac{i}{m}\hat{p}f_2(x_1, x_2, x_3)]\gamma_5 C\}_{\alpha\beta}u_\lambda(p)_\gamma \\ &\quad + \{f_1(x_1, x_2, x_3) - f_2(x_1, x_2, x_3)\}C_{\alpha\beta}\{\gamma_5 u_\lambda(p)\}_\gamma, \\ \bar{\Gamma}_{\alpha\beta,\gamma}(p)_\lambda &= \{C[f_1(-x_1, -x_2, -x_3) + \frac{i}{m}\hat{p}f_2(-x_1, -x_2, -x_3)]\gamma_5\}_{\alpha\beta}\bar{u}_\lambda(p)_\gamma \\ &\quad + \{f_1(-x_1, -x_2, -x_3) - f_2(-x_1, -x_2, -x_3)\}C_{\alpha\beta}\{\bar{u}_\lambda(p)\gamma_5\}_\gamma. \end{aligned} \quad (4)$$

The wave function of $\frac{3}{2}^+$ baryon is

$$\begin{aligned} B_{\alpha\beta\gamma,ijk}^{\frac{3}{2}\lambda,lmn}(x_1, x_2, x_3) &= \frac{1}{2\sqrt{2}}\sqrt{\frac{m}{E}}\varepsilon_{i'j'k'}d_{ijk}^{lmn}\Gamma_{\alpha\beta\gamma}(p)_\lambda, \\ \bar{B}_{\alpha\beta\gamma,ijk}^{\frac{3}{2}\lambda,lmn}(x_1, x_2, x_3) &= \frac{1}{2\sqrt{2}}\sqrt{\frac{m}{E}}\varepsilon_{i'j'k'}d_{ijk}^{lmn}\bar{\Gamma}_{\alpha\beta\gamma}(p)_\lambda. \end{aligned} \quad (5)$$

$$\begin{aligned} \Gamma_{\alpha\beta\gamma}(p)_\lambda &= \{[f_2(x_1, x_2, x_3) - \frac{i}{m}\hat{p}f_1(x_1, x_2, x_3)]\gamma_\mu C\}_{\alpha\beta}\psi_\mu^\lambda(p)_\gamma \\ &\quad + \frac{i}{m}\{f_1(x_1, x_2, x_3) - f_2(x_1, x_2, x_3)\}\{\gamma_\mu\hat{p}\gamma_5 C\}_{\alpha\beta}\{\gamma_5\psi_\mu^\lambda(p)\}_\gamma, \\ \bar{\Gamma}_{\alpha\beta\gamma}(p)_\lambda &= \{C[f_2(-x_1, -x_2, -x_3) + \frac{i}{m}\hat{p}f_1(-x_1, -x_2, -x_3)]\gamma_\mu\}_{\alpha\beta}\bar{\psi}_\mu^\lambda(p)_\gamma \\ &\quad + \frac{i}{m}\{f_1(-x_1, -x_2, -x_3) - f_2(-x_1, -x_2, -x_3)\} \\ &\quad \times \{C\hat{p}\gamma_\mu\gamma_5\}_{\alpha\beta}\{\bar{\psi}_\mu^\lambda(p)\gamma_5\}_\gamma, \end{aligned} \quad (6)$$

where $i'j'k'$ are color indices, $f_{1,2}(x_1, x_2, x_3)$ are two Lorentz-invariant spacial functions, p is the momentum of baryon. The wave functions(3-6) satisfy $SU(6)$ symmetry in the frame of center of mass[2]. They are s-wave in the rest frame.

For the model with three degenerate states [6], the electric charge operator can be written as

$$Q_{k_1k_2}^{k'_1k'_2} = \delta_{k_11}\delta_{k_21}\delta_{k'_1k'_2} - \delta_{k_1k_2}\delta_{k'_13}\delta_{k'_23} \quad (7)$$

and the following relationship is obtained

$$\frac{1}{6}\varepsilon_{k'_1j'i'}\varepsilon_{k'_2j'i'}Q_{k_1k_2}^{k'_1k'_2} = Q_{k_1k_2}. \quad (8)$$

$Q_{k_1 k_2}$ is the electric charge operator in fractional-charge scheme. Therefore, the matrix elements of electric currents should be the same for both schemes of interger charge and fractional charge of quarks.

By using Eqs.(2,3,5,8) and the method of Ref. [1], the matrix elements of electric currents of $\frac{1}{2}^+$ baryon are obtained

$$\begin{aligned}
& \langle B_{\lambda'}^{\frac{1}{2}}(p_f)_{l_1'} | J_\mu(0) | B_{\lambda}^{\frac{1}{2}}(p_i)_{l_2'} \rangle = -\frac{ie}{2} M^2 Q_{k_1 k_1'} (\gamma_\mu + \frac{\kappa}{2m_p} q_\nu \sigma_{\mu\nu})_{\gamma\gamma'} \\
& \times \int d^4 x_1 d^4 x_2 \bar{B}_{\alpha\beta\gamma,ijk_1}^{\frac{1}{2}\lambda'}(x_1, x_2, 0)_{l_1'}^{\dagger} B_{\gamma'\beta\alpha,k_1'ji}^{\frac{1}{2}\lambda}(0, x_2, x_1)_{l_2'}^{\dagger} \\
& = -\frac{ie}{24} (\frac{mm'}{EE'})^{\frac{1}{2}} \{A_1 I_1 - A_2 I_2\}, \tag{9}
\end{aligned}$$

where M is the rest mass of the quark, m, m' and E, E' are the initial and final mass and energy of the baryon respectively,

$$A_1 = S_p \bar{B} Q B, A_2 = S_p \bar{B} B Q. \tag{10}$$

B, \bar{B} are the SU_3 matrices of the initial and final baryon,

$$\begin{aligned}
I_1 = & -20 \{ D_2(q^2) (1 - \frac{m_+}{5m}) + D_2'(q^2) (1 - \frac{m_+}{5m'}) \\
& + \frac{1}{2mm'} (m_-^2 + q^2 + \frac{\kappa m_+}{5m_p} q^2) D_3(q^2) \} \bar{u}_{\lambda'}(p_f) \gamma_\mu u_\lambda(p_i) \\
& -20 \{ 2D_1(q^2) - (1 - \frac{2m_p}{5\kappa m}) D_2(q^2) - (1 - \frac{2m_p}{5\kappa m'}) D_2'(q^2) \\
& + \frac{1}{2mm'} (m_+^2 + \frac{3}{5} q^2) D_3(q^2) \} \frac{\kappa}{2m_p} \bar{u}_{\lambda'}(p_f) q_\nu \sigma_{\mu\nu} u_\lambda(p_i) \\
& -4i \{ \frac{1}{m} D_2(q^2) - \frac{1}{m'} D_2'(q^2) + \frac{\kappa}{2mm'm_p} (m'^2 - m^2) D_3(q^2) \}
\end{aligned}$$

$$\begin{aligned}
& \times q_\mu \bar{u}_{\lambda'}(p_f) u_\lambda(p_i), \\
I_2 = & 4\{(1 - 2\frac{m_+}{m})D_2(q^2) + (1 - 2\frac{m_+}{m'})D_2'(q^2) \\
& + \frac{1}{2mm'}(m_-^2 + q^2 + 2\frac{\kappa m_+}{m_p}q^2)D_3(q^2)\}\bar{u}_{\lambda'}(p_f)\gamma_\mu u_\lambda(p_i) \\
& - 4\{2D_1(q^2) - (1 - \frac{4m_p}{\kappa m})D_2(q^2) - (1 - \frac{4m_p}{\kappa m'})D_2'(q^2) \\
& + \frac{1}{2mm'}(m_+^2 - 3q^2)D_3(q^2)\}\frac{\kappa}{2m_p}\bar{u}_{\lambda'}(p_f)q_\nu\sigma_{\nu\mu}u_\lambda(p_i) \\
& + 8i\{\frac{1}{m}D_2(q^2) - \frac{1}{m'}D_2'(q^2) + \frac{\kappa(m'^2 - m^2)}{2mm'm_p}D_3(q^2)\} \\
& \times q_\mu \bar{u}_{\lambda'}(p_f) u_\lambda(p_i),
\end{aligned} \tag{11}$$

where

$$q_\mu = p_{i\mu} - p_{f\mu}, m_+ = m + m', m_- = m' - m, \tag{12}$$

$$\begin{aligned}
D_1(q^2) &= -M^2 \int f_1'(-x_1, -x_2, 0) f_1(0, x_2, x_1) d^4x_1 d^4x_2, \\
D_2(q^2) &= -M^2 \int f_1'(-x_1, -x_2, 0) f_2(0, x_2, x_1) d^4x_1 d^4x_2, \\
D_2'(q^2) &= -M^2 \int f_2'(-x_1, -x_2, 0) f_1(0, x_2, x_1) d^4x_1 d^4x_2, \\
D_3(q^2) &= -M^2 \int f_2'(-x_1, -x_2, 0) f_2(0, x_2, x_1) d^4x_1 d^4x_2.
\end{aligned} \tag{13}$$

m, m' are the rest mass of the initial and final baryon. $f_j'(-x_1, -x_2, 0)$, $f_j(0, x_2, x_1)$ are the spacial part of the initial and final wave function respectively. Eq. (13) shows that when $pf_\mu \longleftrightarrow pi_\mu$ is taken, we have

$$D_2(q^2) \longleftrightarrow D_2'(q^2), \tag{14}$$

therefore when $m = m'$

$$D_2(q^2) = D_2'(q^2). \quad (15)$$

Similarly, the current matrix elements of $\frac{1}{2}^+$ baryon $\rightarrow \frac{2}{3}^+$ baryon are obtained

$$\begin{aligned} & \langle B_{\lambda'}^{\frac{3}{2}}(p_f)^{lmn} | J_\mu(0) | B_{\lambda}^{\frac{1}{2}}(p_i)_{l_1}^{l_1'} \rangle = -\frac{ie}{2} M^2 Q_{kk'} (\gamma_\mu + \frac{\kappa}{2m_p} q_\nu \sigma_{\mu\nu})_{\gamma\gamma'} \\ & \quad \times \int d^4x_1 d^4x_2 \bar{B}_{\alpha\beta\gamma,ijk}^{\frac{3}{2}\lambda',lmn}(x_1, x_2, 0) B_{\gamma'\beta\alpha,k'ji}^{\frac{1}{2}\lambda}(0, x_2, x_1)_{l_1}^{l_1'} \\ & = \frac{ie}{4} (\frac{mm'}{EE'}) d_{l_1jk}^{lmn} \varepsilon_{jk'l_1'} Q_{kk'} \{ 2D_2(q^2) + \kappa [\frac{m_+}{m_p} D_3(q^2) + 2\frac{m}{m_p} D_1(q^2) \\ & \quad - \frac{m}{m_p} D_2(q^2) - \frac{m}{m_p} D_2'(q^2)] \} \frac{1}{mm'} p_\rho q_\sigma \varepsilon_{\rho\sigma\nu\mu} \bar{\psi}_\nu^{\lambda'}(p_f) u_\lambda(p_i) \\ & \quad + ie (\frac{mm'}{EE'})^{\frac{1}{2}} d_{l_1jk}^{lmn} \varepsilon_{jk'l_1'} Q_{kk'} \{ D_3(q^2) - D_2(q^2) + \frac{\kappa m}{2m_p} [D_2(q^2) \\ & \quad + D_2'(q^2) - 2D_1(q^2)] \} \frac{1}{mm'} (p_{f\mu} q_\nu - p_f \cdot q \delta_{\mu\nu}) \bar{\psi}_\nu^{\lambda'}(p_f) \gamma_5 u_\lambda(p_i) \\ & \quad + ie (\frac{mm'}{EE'})^{\frac{1}{2}} d_{l_1jk}^{lmn} \varepsilon_{jk'l_1'} Q_{kk'} \{ D_2'(q^2) - \frac{m'}{m} D_2(q^2) + \frac{m_-}{m} D_3(q^2) \} \\ & \quad \times \bar{\psi}_\mu^{\lambda'}(p_f) \gamma_5 u_\lambda(p_i). \end{aligned} \quad (16)$$

m, m' are the rest mass of $\frac{1}{2}^+$ baryon and $\frac{2}{3}^+$ baryon respectively,

$$p_\mu = p_{i\mu} + p_{f\mu}. \quad (17)$$

In Eq.(11), when $m = m'$ is taken, the terms in I_1 and I_2 , which are proportional to q_μ vanish. Thus, when $m = m'$, the current matrix element of $\frac{1}{2}^+$ baryon automatically satisfies current conservation. In general, in order to satisfy current conservation, the following

relationship must be satisfied

$$D_2'(q^2) - \frac{m'}{m}D_2(q^2) + \frac{m_-}{m}D_3(q^2) = 0. \quad (18)$$

For $\frac{1}{2}^+$ baryons the only matrix element with $m' \neq m$ is $\Sigma^0 \rightarrow \Lambda$. For this process, we have

$$A_1 = A_2 = \frac{1}{2\sqrt{3}}. \quad (19)$$

The condition(18) guarantees current conservation.

3 Relationship Between $f_1(x_1, x_2, x_3)$ and $f_2(x_1, x_2, x_3)$

In this section, we study the behavior of two invariant spacial functions $f_1(x_1, x_2, x_3)$ and $f_2(x_1, x_2, x_3)$ in the frame of center-of-mass. $\Gamma_{\alpha\beta,\gamma}(p)_\lambda(4)$ can be written as

$$\begin{aligned} \Gamma_{\alpha\beta,\gamma}(x_1, x_2, x_3)_\lambda &= g_1(x_1, x_2, x_3)\{(1 + \gamma_4)\gamma_5 C\}_{\alpha\beta}u_{\lambda,\gamma} \\ &\quad + g_2(x_1, x_2, x_3)\{[(1 - \gamma_4)\gamma_5 C]_{\alpha\beta}u_{\lambda,\gamma} + 2C_{\alpha\beta}(\gamma_5 u_\lambda)_\gamma\}, \\ g_1(x_1, x_2, x_3) &= \frac{1}{2}\{f_1(x_1, x_2, x_3) + f_2(x_1, x_2, x_3)\}, \\ g_2(x_1, x_2, x_3) &= \frac{1}{2}\{f_1(x_1, x_2, x_3) - f_2(x_1, x_2, x_3)\}, \end{aligned} \quad (20)$$

x_1, x_2, x_3 are the time-space coordinates of three quarks. In Ref.[1], in order to guarantee SU_6 symmetry, it is assumed that strong interaction satisfies SU_6 symmetry when the speed of the quark is much less than the speed of light. One possibility is that strong interaction

takes scalar form. The B-S equation of the baryon is written as

$$\begin{aligned}
& (i\hat{p}_1 + M)_{\alpha\alpha'} (i\hat{p}_2 + M)_{\beta\beta'} (i\hat{p}_3 + M)_{\gamma\gamma'} B_{\alpha'\beta'\gamma',ijk}^{\frac{1}{2}\lambda}(p_1, p_2, p_3)_l^m \\
= & -i(i\hat{p}_3 + M)_{\gamma\gamma'} \int U(q) B_{\alpha\beta\gamma',ijk}^{\frac{1}{2}\lambda}(p_1 - q, p_2 + q, p_3)_l^m d^4q \\
& -i(i\hat{p}_1 + M)_{\alpha\alpha'} \int U(q) B_{\alpha'\beta\gamma,ijk}^{\frac{1}{2}\lambda}(p_1, p_2 - q, p_3 + q)_l^m d^4q \\
& -i(i\hat{p}_2 + M)_{\beta\beta'} \int U(q) B_{\alpha\beta'\gamma,ijk}^{\frac{1}{2}\lambda}(p_1 + q, p_2, p_3 - q)_l^m d^4q \\
& - \int V(q_1, q_2, q_3) \delta^4(q_1 + q_2 + q_3) B_{\alpha\beta\gamma,ijk}^{\frac{1}{2}\lambda}(p_1 + q_1, p_2 + q_2, p_3 + q_3) \\
& \times d^4q_1 d^4q_2 d^4q_3.
\end{aligned} \tag{21}$$

$B_{\alpha\beta\gamma,ijk}^{\frac{1}{2}\lambda}(p_1, p_2, p_3)_l^m$ is the wave function for $\frac{1}{2}^+$ baryon in the frame of center-of-mass. We assume $U(q)$ and $V(q_1, q_2, q_3)$ are independent of the momentum of the baryon.

$$p_1 + p_2 + p_3 = p, \tag{22}$$

where p is the momentum of $\frac{1}{2}^+$ baryon.

According to Ref.[2], in order to satisfy SU_6 symmetry the terms at $O(\frac{|\mathbf{p}_j|}{M})(j = 1, 2, 3)$ in the wave function are ignored. The same treatment is used in Eq.(21). Substituting the wave function of $\frac{1}{2}^+$ into Eq.(21), we obtain

$$\begin{aligned}
& (M - \gamma_4 p_{10})_{\alpha\alpha'} (M - \gamma_4 p_{20})_{\beta\beta'} (M - \gamma_4 p_{30})_{\gamma\gamma'} \Gamma_{\alpha'\beta'\gamma'}(p_1, p_2, p_3)_\lambda \\
= & -i(M - \gamma_4 p_{30})_{\gamma\gamma'} \int U(q) \Gamma_{\alpha\beta,\gamma'}(p_1 - q, p_2 + q, p_3)_\lambda d^4q \\
& -i(M - \gamma_4 p_{10})_{\alpha\alpha'} \int U(q) \Gamma_{\alpha'\beta,\gamma}(p_1, p_2 - q, p_3)_\lambda d^4q
\end{aligned}$$

$$\begin{aligned}
& -i(M - \gamma_4 p_{20})_{\beta\beta'} \int U(q) \Gamma_{\alpha\beta',\gamma}(p_1 + q, p_2, p_3 - q)_\lambda d^4 q \\
& - \int V(q_1, q_2, q_3) \delta^4(q_1 + q_2 + q_3) \Gamma_{\alpha\beta,\gamma}(p_1 + q_1, p_2 + q_2, p_3 + q_3) \\
& \times d^4 q_1 d^4 q_2 d^4 q_3.
\end{aligned} \tag{23}$$

where $\Gamma_{\alpha\beta,\gamma}(p_1, p_2, p_3)_\lambda$ is the expression of $\Gamma_{\alpha\beta,\gamma}(x_1, x_2, x_3)_\lambda$ (20) in the momentum representation. Calculations lead to

$$\begin{aligned}
& (M - p_{10})(M - p_{20})(M - p_{30}) g_1(p_1, p_2, p_3) \\
= & -i \int U(q) \{ (M - p_{30}) g_1(p_1 - q, p_2 + q, p_3) \\
& + (M - p_{10}) g_1(p_1, p_2 - q, p_3 + q) \\
& + (M - p_{20}) g_1(p_1 + q, p_2, p_3 - q) \} d^4 q \\
& - \int V(q_1, q_2, q_3) \delta^4(q_1 + q_2 + q_3) \\
& \times g_1(p_1 + q_1, p_2 + q_2, p_3 + q_3) d^4 q_1 d^4 q_2 d^4 q_3,
\end{aligned} \tag{24}$$

$$\begin{aligned}
& (M + p_{10})(M + p_{20})(M - p_{30}) g_2(p_1, p_2, p_3) \\
= & -i \int U(q) \{ (M - p_{30}) g_2(p_1 - q, p_2 + q, p_3) \\
& + (M + p_{10}) g_2(p_1, p_2 - q, p_3 + q) \\
& + (M + p_{20}) g_2(p_1 + q, p_2, p_3 - q) \} d^4 q \\
& - \int V(q_1, q_2, q_3) \delta^4(q_1 + q_2 + q_3) \\
& \times g_2(p_1 + q_1, p_2 + q_2, p_3 + q_3) d^4 q_1 d^4 q_2 d^4 q_3,
\end{aligned} \tag{25}$$

$$\begin{aligned}
& (M + p_{10})(M - p_{20})(M + p_{30})g_2(p_1, p_2, p_3) \\
= & -i \int U(q) \{ (M + p_{30})g_2(p_1 - q, p_2 + q, p_3) \\
& + (M + p_{10})g_2(p_1, p_2 - q, p_3 + q) \\
& + (M - p_{20})g_2(p_1 + q, p_2, p_3 - q) \} d^4 q \\
& - \int V(q_1, q_2, q_3) \delta^4(q_1 + q_2 + q_3) \\
& \times g_2(p_1 + q_1, p_2 + q_2, p_3 + q_3) d^4 q_1 d^4 q_2 d^4 q_3, \tag{26}
\end{aligned}$$

$$\begin{aligned}
& (M - p_{10})(M + p_{20})(M + p_{30})g_2(p_1, p_2, p_3) \\
= & -i \int U(q) \{ (M + p_{30})g_2(p_1 - q, p_2 + q, p_3) \\
& + (M - p_{10})g_2(p_1, p_2 - q, p_3 + q) \\
& + (M + p_{20})g_2(p_1 + q, p_2, p_3 - q) \} d^4 q \\
& - \int V(q_1, q_2, q_3) \delta^4(q_1 + q_2 + q_3) \\
& \times g_2(p_1 + q_1, p_2 + q_2, p_3 + q_3) d^4 q_1 d^4 q_2 d^4 q_3. \tag{27}
\end{aligned}$$

Since $V(q_1, q_2, q_3)$ are totally symmetric functions of q_1, q_2, q_3 . $g_1(p_1, p_2, p_3)$ are totally symmetric functions of p_1, p_2, p_3 , which is consistent with Ref.[2]. From Eqs.(25-27), we see that $g_2(p_1, p_2, p_3)$ have following symmetries: (1) totally symmetric in p_1, p_2, p_3 . (2) since $U(q)$ and $V(q_1, q_2, q_3)$ are independent of the momentum p , the equation is invariant under the transformations $p_{20} \rightarrow -p_{20}, p_{30} \rightarrow -p_{30}; p_{10} \rightarrow -p_{10}, p_{30} \rightarrow -p_{30}; p_{10} \rightarrow -p_{10}, p_{20} \rightarrow -p_{20}$.

By using the second symmetry of $g_2(p_1, p_2, p_3)$, Eq.(3.6) becomes Eq.(3.5) under the transformation $p_{10} \rightarrow -p_{10}$, $p_{20} \rightarrow -p_{20}$, thus $g_1(p_1, p_2, p_3)$ and $g_2(p_1, p_2, p_3)$ satisfy the same equation. $g_1(p_1, p_2, p_3)$ is related to $g_2(p_1, p_2, p_3)$ by

$$g_1(p_1, p_2, p_3) = b g_2(p_1, p_2, p_3), \quad (28)$$

where b is a constant. Eq.(28) leads to

$$f_2(x_1, x_2, x_3) = a f_1(x_1, x_2, x_3). \quad (29)$$

Thus, in the wave functions (3,5), there is only one independent spacial function.

Substituting Eq.(29) into Eq.(18), we obtain

$$a = \frac{1}{1 - \frac{m_0}{m}} \quad \text{or} \quad a = 1. \quad (30)$$

m_0 is a parameter, m is the physical mass of the baryon. Generally $a \neq 1$, a takes the first expression of Eq.(30).

4 Electromagnetic Properties of $\frac{1}{2}^+$ Baryons

The electromagnetic form factors of $\frac{1}{2}^+$ baryon are obtained from the current matrix elements Eq.(9,11)

$$G_E(q^2) = -\frac{2}{3}(A_1 + 2A_2)\left(1 + \frac{q^2}{4m^2}\right)\left\{D_2(q^2) - \frac{\kappa q^2}{4mm_p}D_3(q^2)\right\}$$

$$\begin{aligned}
& +\frac{1}{3}(A_2 + 5A_1)\{D_2(q^2) + \frac{q^2}{4m^2}[D_3(q^2) + \frac{\kappa m}{m_p}D_2(q^2) \\
& -\frac{\kappa m}{m_p}D_1(q^2) - \kappa\frac{m}{m_p}(1 + \frac{q^2}{4m^2})D_3(q^2)]\}.
\end{aligned} \tag{31}$$

$$\begin{aligned}
G_M(q^2) = & \frac{1}{3}(A_2 + 5A_1)\{D_2(q^2) + \frac{q^2}{4m^2}D_3(q^2) + \kappa\frac{m}{m_p}[D_1(q^2) \\
& -D_2(q^2) + (1 + \frac{q^2}{4m^2})D_3(q^2)]\},
\end{aligned} \tag{32}$$

where m is the mass of the baryon. From Eq.(31) we obtain

$$D_2(0) = 1. \tag{33}$$

The expression of the magnetic moment of $\frac{1}{2}^+$ baryon is obtained from Eq.(32)

$$\mu = \frac{1}{3}(A_2 + 5A_1)\{\frac{m_p}{m} + \kappa[D_1(0) + D_3(0) - 1]\}. \tag{34}$$

From Eqs.(40,41,33)

$$\mu = \frac{1}{3}(A_2 + 5A_1)\{\frac{m_p}{m} + \kappa(\frac{1}{1 - \frac{m_0}{m}} - \frac{m_0}{m})\} \tag{35}$$

is obtained. The two parameters κ, m_0 in Eq.(35) are determined to be

$$\kappa = 0.481, m_0 = 0.778m_p \tag{36}$$

by input the magnetic moments of proton and Σ [8]. The magnetic moments of other six baryons are determined to be

	μ_p	μ_n	μ_Λ	μ_{Σ^+}	μ_{Σ^0}	μ_{Σ^-}	μ_{Ξ^0}	μ_{Ξ^-}
theory	2.79	-1.86	-0.64	1.74	0.58	-0.57	-0.97	-0.51
	(input)		(input)					
exp	2.79	-1.91	-0.67	2.59				-1.93
			± 0.46	± 0.46				± 0.75

The electromagnetic form factors of proton and neutron are found from Eq.(31,32)

$$\begin{aligned}
G_E^p(q^2) &= D_2(q^2) + \frac{q^2}{4m_N^2} \{D_3(q^2) + \kappa[D_2(q^2) - D_1(q^2) - (1 + \frac{q^2}{4m_N^2})D_3(q^2)]\}, \\
G_M^p(q^2) &= D_2(q^2) + \kappa[D_1(q^2) + D_3(q^2) - D_2(q^2)] + (1 + \kappa)\frac{q^2}{4m_N^2}D_3(q^2), \tag{37}
\end{aligned}$$

$$\begin{aligned}
G_E^n(q^2) &= -\frac{2}{3}\frac{q^2}{4m_N^2} \{D_3(q^2) - D_2(q^2) + \kappa[D_2(q^2) - D_1(q^2)]\}, \\
G_M^n(q^2) &= -\frac{2}{3}G_M^p(q^2). \tag{38}
\end{aligned}$$

By using Eqs.(29,30,36)

$$\begin{aligned}
G_E^p(q^2) &= D_2(q^2)\{1 + \tau(2.71 - 2.17\tau)\}, \\
G_M^p(q^2) &= \mu_p D_2(q^2)\{1 + 2.39\tau\}, \\
G_E^n(q^2) &= 1.39\mu_n\tau D_2(q^2) \tag{39}
\end{aligned}$$

are obtained, where $\tau = \frac{q^2}{4m_N^2}$. It is seen from Eq.(39) that there is an invariant function $D_2(q^2)$ in the three form factors, which can be determined from the experimental data of the

magnetic form factor of proton [9]

$$D_2(q^2) = \frac{1}{(1 + \frac{q^2}{0.71})^2(1 + 2.39\tau)}. \quad (40)$$

The ratio of the electric and magnetic form factor of proton is obtained

$$\frac{\mu_p G_E^p(q^2)}{G_M^p(q^2)} = \frac{1 + \tau(2.71 - 2.17\tau)}{1 + 2.39\tau}. \quad (41)$$

Comparisons with data are shown in Fig.1 and 2. The experimental data of Fig.1 is from Ref.[10], and that for Fig.2 is from Ref.[11].

The expression of the electric form factor of neutron is obtained

$$G_E^n(q^2) = 1.39\tau G_M^n(q^2)(1 + 2.39\tau)^{-1}. \quad (42)$$

The slope of $G_E^n(q^2)$ at $q^2 = 0$ is

$$\left. \frac{dG_E^n(q^2)}{dq^2} \right|_{q^2=0} = 1.39 \frac{\mu_n}{4m_N^2} = -0.73[GeV]^{-2}. \quad (43)$$

The experimental data are

$$-0.579 \pm 0.018^{[12]}, -0.512 \pm 0.049^{[13]}, 0.495 \pm 0.010^{[14]}. \quad (44)$$

Comparisons of Eq.(42) with the experimental data are shown in Fig.3 and Fig.4. The experimental data of Fig.3 comes from Ref.[4] and that for Fig.4 comes from Ref.[11].

At $q^2 = -4m_N^2$, there are

$$\begin{aligned} G_E^p(-4m^2) &= G_M^p(-4m^2) = 0.18, \\ G_E^n(-4m^2) &= G_M^p(-4m^2) = -\frac{2}{3}G_E^p(-4m^2). \end{aligned} \quad (45)$$

The S-matrix element of $\Sigma^0 \rightarrow \Lambda + \gamma$ is studied

$$\begin{aligned}
\langle \gamma \Lambda | S | \Sigma^0 \rangle &= -ie(2\pi)^4 \delta(p_i - p_f - q) \frac{e_\mu^\lambda}{\sqrt{2\omega}} \left(\frac{m_\Lambda}{E_\Lambda} \right)^{\frac{1}{2}} \mu_{\Sigma_0 \Lambda} \\
&\quad \times \frac{\kappa}{2m_p} \bar{u}_{\lambda'}(p_f) q_\nu \sigma_{\nu\mu} u_\lambda(p_i),
\end{aligned} \tag{46}$$

$$\mu_{\Sigma_0 \Lambda} = \frac{1}{2\sqrt{3}} D_3(0) \left\{ \frac{2}{a_\Lambda a_{\Sigma^0}} - \frac{1}{a_\Lambda} \left(1 - \frac{m_p}{\kappa m_\Sigma} \right) - \frac{1}{a_{\Sigma^0}} \left(1 - \frac{m_p}{\kappa m_\lambda} \right) + \frac{m_+^2}{2m_\Lambda m_\Sigma} \right\}. \tag{47}$$

The dependences of $D_1(0)$, $D_2(0)$, $D_1'(0)$ and $D_2'(0)$ on the mass of initial and final baryon need to be found. From Eq.(28) we have

$$\frac{D_2(0)}{D_2'(0)} = \frac{a}{a'}. \tag{48}$$

On the other hand, Eq.(14) shows when $m \longleftrightarrow m'$ is taken, we have

$$D_2(0) \longleftrightarrow D_2'(0). \tag{49}$$

When $m = m'$, Eqs.(15,33) lead to

$$D_2(0) = D_2'(0) = 1. \tag{50}$$

The general expressions of $D_2(0)$, $D_2'(0)$ which satisfy Eqs.(48-50) are found

$$\begin{aligned}
D_2(0) &= \left(\frac{a}{a'} \right)^{\frac{1}{2}} f(m, m'), \\
D_2'(0) &= \left(\frac{a'}{a} \right)^{\frac{1}{2}} f(m, m').
\end{aligned} \tag{51}$$

$f(m, m')$ is a symmetric function of m , m' and

$$f(m, m) = 1. \quad (52)$$

When $m \neq m'$, the deviation of $f(m, m')$ from 1 is proportional to $(m - m')^2$. According to Ref.[1], $f(m, m')$ is the effect of Lorentz contraction. We obtain

$$f(m, m') = \frac{4mm'}{(m + m')^2}. \quad (53)$$

For $\Sigma^0 \rightarrow \Lambda + \gamma$, the deviation of $f(m, m')$ from 1 is only 0.1%. From Eq.(51), the expressions of $D_1(0)$ and $D_3(0)$ are found

$$\begin{aligned} D_3(0) &= \sqrt{aa'} f(m, m'), \\ D_1(0) &= \frac{1}{\sqrt{aa'}} f(m, m'). \end{aligned} \quad (54)$$

The magnetic moment of $\Sigma \rightarrow \Lambda$ and the decay rate are computed to be

$$\mu_{\Sigma^0 \Lambda} = 1.053 \quad (55)$$

$$\begin{aligned} \Gamma &= \frac{\alpha}{8} \mu_{\Sigma^0 \Lambda}^2 \frac{m_\Sigma^3}{m_p^2} \left(1 - \frac{m_\Lambda^2}{m_\Sigma^2}\right)^3 = 3.79 \times 10^{-3} MeV, \\ \tau &= \frac{1}{\Gamma} = 1.74 \times 10^{-19} sec. \end{aligned} \quad (56)$$

The experimental upper limit is

$$\tau < 1.0 \times 10^{-14} sec. \quad (57)$$

5 Electromagnetic transition of $p \rightarrow \Delta(1236)$

The matrix elements of currents are obtained from Eqs.(18,28,16). Substituting Eqs.(18),(28) into Eq.(16), we derive

$$\begin{aligned}
< \Delta_{\lambda'}^+(p_f) | J_\mu(0) | p_\lambda(p_i) > = & -\frac{ie}{4\sqrt{3}} \left(\frac{mm'}{EE'}\right)^{\frac{1}{2}} A \frac{1}{mm'} D_3(q^2) p_\rho q_\sigma \\
& \times \varepsilon_{\rho\sigma\nu\mu} \bar{\psi}_\nu^{\lambda'}(p_f) u_\lambda(p_i) - \frac{ie}{\sqrt{3}} \left(\frac{mm'}{EE'}\right)^{\frac{1}{2}} \frac{B}{mm'} D_3(q^2) \\
& \times (p_{f\mu} q_\nu - p_f \cdot q \delta_{\mu\nu}) \bar{\psi}_\nu^{\lambda'}(p_f) \gamma_5 u_\lambda(p_i),
\end{aligned} \tag{58}$$

where

$$\begin{aligned}
A &= \frac{2}{a'} + \kappa \left\{ 1 + \frac{m'}{m_p} + \frac{2}{aa'} - \frac{1}{a} - \frac{1}{a'} \right\}, \\
B &= 1 - \frac{1}{a'} + \frac{\kappa}{2} \left\{ \frac{1}{a} + \frac{1}{a'} - \frac{2}{aa'} \right\},
\end{aligned} \tag{59}$$

and

$$A = 1.717, B = 0.699. \tag{60}$$

The S matrix element of $\gamma p \rightarrow \pi N$ is written as

$$\begin{aligned}
< \pi N | S | \gamma p > &= -i(2\pi)^4 \delta(p_\gamma + p_i - p_\pi - p_N) \sum_{\lambda'} < \pi N | U | \Delta_{\lambda'}^+(p_f) > \\
&\times < \Delta_{\lambda'}^+(p_f) | U | \gamma p > \frac{E_\Delta}{m_\Delta} \frac{1}{W - m_\Delta + \frac{i}{2}\Gamma(W)},
\end{aligned} \tag{61}$$

where W is the mass of the final state, $\Gamma(W)$ is the total width of the strong decay of $\Delta(1236)$. The calculation is done in the rest frame of $\Delta(1236)$. $< \pi N | U | \Delta_{\lambda'}^+(p_f) >$ is the

amplitude of the strong decay of $\Delta(1236)$

$$\langle \pi N | U | \Delta_{\lambda'}^+(p_f) \rangle = \left(\frac{m_N}{2E_\pi E_N} \right)^{\frac{1}{2}} g(W) \frac{p_{\pi\mu}}{m_N} \bar{u}(p_N) \psi_\mu^{\lambda'}. \quad (62)$$

The electric transition amplitude in Eq.(60) is expressed as

$$\langle \Delta_{\lambda'} | U | \gamma p \rangle = -\frac{1}{\sqrt{2E_\gamma}} e_\mu \langle \Delta_{\lambda'} | J_\mu(0) | p \rangle. \quad (63)$$

By using following equation

$$\begin{aligned} \sum_{\lambda'} \psi_\mu^{\lambda'} \bar{\psi}_{\mu'}^{\lambda'} &= \frac{1}{3} (1 + \gamma_4) \{ \delta_{\mu\mu'} + \frac{1}{2} \gamma_5 \gamma_j \varepsilon_{j\mu\mu'} \} \\ (j, \mu, \mu' &= 1, 2, 3) \end{aligned} \quad (64)$$

and Eq.(58), we obtain

$$\begin{aligned} \sum_{\lambda'} \bar{u}_\gamma(p_N) \psi_\mu^{\lambda'} &< \Delta_{\lambda'} | J_\nu(0) | p_\lambda \rangle p_{\pi\mu} e_\nu \\ &= \frac{eD_3(0)}{24\sqrt{3}m_N^2 m_\Delta} \left\{ \frac{m_N(m_N + E_N)}{E_i(m_N + E_i)} \right\}^{\frac{1}{2}} \bar{u}_\gamma \{ [A(m_N + m_\Delta)^2 + B(m_\Delta^2 - m_N^2)] \\ &\quad \times [2\mathbf{k} \cdot (\mathbf{e} \cdot \mathbf{p}_\pi) + i\sigma \cdot \mathbf{e} \mathbf{p}_\pi \cdot \mathbf{k} - i\sigma \cdot \mathbf{k} \mathbf{p}_\pi \cdot \mathbf{e}] \\ &\quad - 3iB(m_\Delta^2 - m_N^2)(\sigma \cdot \mathbf{e} \mathbf{p}_\pi \cdot \mathbf{k} + \sigma \cdot \mathbf{k} \mathbf{p}_\pi \cdot \mathbf{e}) \} u_\lambda, \end{aligned} \quad (65)$$

where E_i is the energy of the initial proton, k is the energy of the photon. The amplitudes of the magnetic and electric transitions are obtained by comparing with the photo production amplitudes in Ref.[15]

$$M1+ = \frac{eD_3(0)}{96\sqrt{3}\pi m_N^2 m_\Delta} \left\{ \frac{m_N + E_N}{m_\Delta E_i(m + E_i)} \right\}^{\frac{1}{2}} \frac{g(W)p_\pi k}{W - m_\Delta + \frac{i}{2}\Gamma(W)}$$

$$\times \{A(m_N + m_\Delta)^2 + B(m_0^2 - m_N^2)\}, \quad (66)$$

$$\begin{aligned} E1+ &= -\frac{eD_3(0)}{96\sqrt{3}\pi m_N^2 m_\Delta} \left\{ \frac{m_N + E_N}{m_\Delta E_i(m + E_i)} \right\}^{\frac{1}{2}} \frac{g(W)p_\pi k}{W - m_\Delta + \frac{i}{2}\Gamma(W)} \\ &\times B(m_\Delta^2 - m_N^2), \end{aligned} \quad (67)$$

$$\frac{E1+}{M1+} = \frac{-B(m_\Delta - m_N)}{A(m_\Delta + m_N) + B(m_\Delta - m_N)} = -5.4\%. \quad (68)$$

There are several experimental values: -0.045 [16], -0.073 [17], -0.024 [18].

The partial width of $\Delta^+(1236) \rightarrow p + \gamma$ is derived

$$\Gamma_\gamma = \frac{k^2 m_N}{2\pi m_\Delta} \{ |A_{\frac{3}{2}}|^2 + |A_{\frac{1}{2}}|^2 \}, \quad (69)$$

$$\begin{aligned} A_{\frac{3}{2}} &= -\frac{eD_3(0)(m_\Delta + m_N)(m_\Delta^2 - m_N^2)}{8\sqrt{6}(m_N m_\Delta)^{3/2}} \left\{ A + 2B \frac{m_\Delta - m_N}{m_\Delta + m_N} \right\} \\ &= -0.21[GeV]^{-\frac{1}{2}}, \\ A_{\frac{1}{2}} &= -\frac{eD_3(0)(m_\Delta + m_N)(m_\Delta^2 - m_N^2)}{24\sqrt{2}(m_N m_\Delta)^{3/2}} \left\{ A - 2B \frac{m_\Delta - m_N}{m_\Delta + m_N} \right\} \\ &= -0.10[GeV]^{-\frac{1}{2}}. \end{aligned} \quad (70)$$

The experimental data are [19]

$$A_{\frac{3}{2}} = -0.24[GeV]^{-\frac{1}{2}}, A_{\frac{1}{2}} = -0.14[GeV]^{-\frac{1}{2}}. \quad (71)$$

The decay width is computed to be

$$\Gamma_\gamma = 0.64 MeV, \quad (72)$$

The experimental data [21] is 0.65MeV.

6 Magnetic moment and electromagnetic form factors

of $p \rightarrow \Delta^+(1236)$

The differential cross section of the electric production

$$e + p \rightarrow e + \Delta^+(1236)$$

$$\hookrightarrow N + \pi$$

is expressed as

$$\frac{1}{\Gamma_t} \frac{d^2\sigma}{d\Omega dE'} = \sigma_T + \varepsilon\sigma_S. \quad (73)$$

where E' is the energy of the outgoing electron. Use of the equation

$$\begin{aligned} \sum_{\lambda} \psi_{\mu}^{\lambda}(p) \bar{\psi}_{\mu'}^{\lambda}(p) &= \frac{1}{2} \left(1 - \frac{i}{m'} \hat{p}\right) \left\{ \delta_{\mu\mu'} + \frac{2}{3} \frac{p_{\mu} p_{\mu'}}{m'^2} - \frac{1}{3} \gamma_{\mu} \gamma_{\mu'} \right. \\ &\quad \left. - \frac{i}{3m'} (p_{\mu} \gamma_{\mu'} - p_{\mu'} \gamma_{\mu}) \right\} \end{aligned} \quad (74)$$

and Eq.(58) leads to

$$\begin{aligned} \sigma_T &= \frac{m\alpha q^{*2}}{m'(W^2 - m^2)} \frac{\Gamma(W)}{(W - m')^2 + \frac{1}{4}\Gamma^2(W)} \frac{D_3^2(q^2)}{18m^2} \left\{ A^2(q^2 + m_+^2) \right. \\ &\quad \left. + 2AB(m'^2 - m^2 - q^2) + 4B^2(q^2 + m_-^2) \left(1 - \frac{q^2}{q^{*2}}\right) \right\}, \end{aligned} \quad (75)$$

$$\sigma_S = \frac{m\alpha q^{*2}}{m'(W^2 - m^2)} \frac{\Gamma(W)}{(W - m')^2 + \frac{1}{4}\Gamma^2(W)} \frac{2D_3^2(q^2)}{9m^2} B^2(q^2 + m_+^2) \frac{q^2}{q^{*2}}, \quad (76)$$

where

$$W^2 = -(p_i + p_e - p_{e'})^2, q^{*2} = q^2 + \frac{1}{4m'^2} (m'^2 - m^2 - q^2)^2. \quad (77)$$

The ratio of σ_S and σ_T is obtained

$$R = \frac{\sigma_S}{\sigma_T} = [4B^2(q^2 + m_-^2) \frac{q^2}{q^{*2}}] / [(Am_+ + Bm_-)^2 + (A - B)^2 q^2 + 3B^2(q^2 + m_-^2) - 4B^2(q^2 + m_-^2) \frac{q^2}{q^{*2}}]. \quad (78)$$

The behavior of R is released

$$\begin{aligned} q^2 &= 0, & R &= 0; \\ q^2 &\rightarrow \infty, & R &\sim \frac{1}{q^2} \rightarrow 0. \end{aligned} \quad (79)$$

In the range of $q^2 > 3[GeV]^2$, $R \sim 0.27$.

According to the definition of multipoles, the magnetic transition form factor $G_{M1+}^2(q^2)$, the electric transition form factor $G_{E1+}^2(q^2)$ and $G_{S1+}^2(q^2)$ are found

$$G_{M1+}^2(q^2) = \frac{D_3^2(q^2)}{18m^2} \{ (Am_+ + Bm_-)^2 + (A - B)q^2 - B^2(q^2 + m_-^2) \frac{q^2}{q^{*2}} \}, \quad (80)$$

$$G_{E1+}^2(q^2) = \frac{D_3^2(q^2)}{18m^2} B^2(q^2 + m_-^2) (1 - \frac{q^2}{q^{*2}}), \quad (81)$$

$$G_{S1+}^2(q^2) = \frac{D_3^2(q^2)}{18m^2} B^2(q^2 + m_-^2). \quad (82)$$

The differential cross section(73) is expressed as

$$\begin{aligned} \frac{1}{\Gamma_t} \frac{d^2\sigma}{dE' d\Omega} &= \frac{m\alpha q^{*2}}{m'(W^2 - m^2)} \{ G_{M1+}^2(q^2) + 3G_{E1+}^2(q^2) \\ &\quad + 4\epsilon G_{S1+}^2(q^2) \frac{q^2}{q^{*2}} \} \frac{\Gamma(W)}{(W - m')^2 + \frac{1}{4}\Gamma^2(W)}. \end{aligned} \quad (83)$$

From Eq.(79), the magnetic moment of $p \rightarrow \Delta^+(1236)$ is derived

$$\mu = G_{M1+}(0) = \frac{D_3(0)}{3\sqrt{2}m} (Am_+ + Bm_-)^2 = 1.23 \frac{2\sqrt{2}}{3} \mu_p. \quad (84)$$

The data are

$$1.22\frac{2\sqrt{2}}{3}\mu_p^{[18]}, \quad 1.28\frac{2\sqrt{2}}{3}\mu_p^{[21]}. \quad (85)$$

Lorentz contraction effect is considered in Eq.(84), for $p \rightarrow \Delta^+(1236)$

$$f(m, m') = 0.98. \quad (86)$$

If this effect is ignored, the theoretical value of the magnetic transition moment is $1.26\frac{2\sqrt{2}}{3}\mu_p$.

The electric multipole moments are obtained from Eq.(81)

$$E1+ = -\frac{D_3(0)}{3\sqrt{2}m}Bm_- = -0.17, \quad (87)$$

$$S1+ = E1+,$$

$$\frac{S1+}{\mu} = -5.4\%. \quad (88)$$

The data [4] is

$$\frac{S1+}{\mu} = (-5 \pm 3)\%. \quad (89)$$

Theoretical results agree with experimental data.

The expression

$$\sigma_T^R = \frac{m\alpha q^{*2}}{m'(W^2 - m^2)} \frac{\Gamma(W)}{(W - m')^2 + \frac{1}{4}\Gamma^2(W)} G_M^2(q^2) \quad (90)$$

has been used to determine $G_M^2(q^2)$. G_M is obtained from Eq.(75) that

$$\begin{aligned} G_M^2(q^2) &= G_{M1+}^2(q^2) + 3G_{E1+}^2(q^2) \\ &= \frac{D_3^2(q^2)}{18m^2} \{ (Am_+ + Bm_-)^2 + (A - B)^2 q^2 + B^2(q^2 + m_-^2)(3 - 4\frac{q^2}{q^{*2}}) \}. \end{aligned} \quad (91)$$

The mass difference between $\Delta^+(1236)$ and proton is ignored. The integral $D_3(q^2)$ for $p \rightarrow \Delta^+(1236)$ is expressed as

$$D_3(q^2) = \frac{4mm' \sqrt{aa'}}{(m+m')^2} (1 + 2.39 \frac{q^2}{4m^2})^{-1} (1 + \frac{q^2}{0.71})^{-1}. \quad (92)$$

Substituting Eq.(92) into Eq.(91), the expression of $G_M^2(q^2)$ is obtained. Comparisons with experimental data are shown in Fig.5 and 6. The data for Fig.5 comes from Ref.[4] and that for Fig.6 comes from Ref.[22]. It can be seen from these two figures that as q^2 increases, the theoretical curve drops a little bit faster than the experimental one. At $q^2 = 0.8[GeV]^2$, the theoretical value is 10% less than the experimental value. This difference can be regarded as to be from the ignorance of the mass difference between proton and $\Delta^+(1236)$.

Taking

$$W = m' = 1.236 GeV, \quad \Gamma(m') = 0.12 GeV, \quad (93)$$

we obtain

$$\sigma_S = 48.4 q^2 (q^2 + 0.0888) (1 + 0.679 q^2)^{-2} (1 + \frac{q^2}{0.71})^{-4} \times 10^{-28} cm^2. \quad (94)$$

Comparison with the data [23] is shown in Fig.7.

7 Discussion

$SU(6)$ summetric wave functions of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ of s- wave(in the frame of center of mass) are applied to study the electromagnetic form factors of nucleons and $p \rightarrow \Delta$. A new expression

of $\frac{\mu_p G_E^p(q^2)}{G_M^p(q^2)}$ is obtained. Nonzero electric form factor $G_E^n(q^2)$ is found. The magnetic form factor of $p \rightarrow \Delta$ decreases faster than G_M^p . Nonzero multipole moments $E1+$ and $S1+$ are obtained. They are small and negative. It is interesting to point out that nonzero G_E^n , $E1+$, and $S1+$ are resulted in the additional terms of the wave function, which are constructed by the spinors of antiquarks. The amplitudes and decay rate of $\Delta \rightarrow p + \gamma$ are computed and theory agrees with data.

The magnetic moments of hyperons are calculated under the assumption that the anomalous magnetic moment of strange quark(except for the charge factor) is the same as the one of u and d quarks.

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Figure Captions

FIG. 1. Ratio of electric and magnetic form factors of proton.

FIG. 2. Ratio of electric and magnetic form factors of proton.

FIG. 3. Electric form factor of neutron.

FIG. 4. Electric form factor of neutron.

FIG. 5. Magnetic form factor of $p \rightarrow \Delta$.

FIG. 6. Magnetic form factor of $p \rightarrow \Delta$.

FIG. 7. Cross Section of virtual scalar photon.

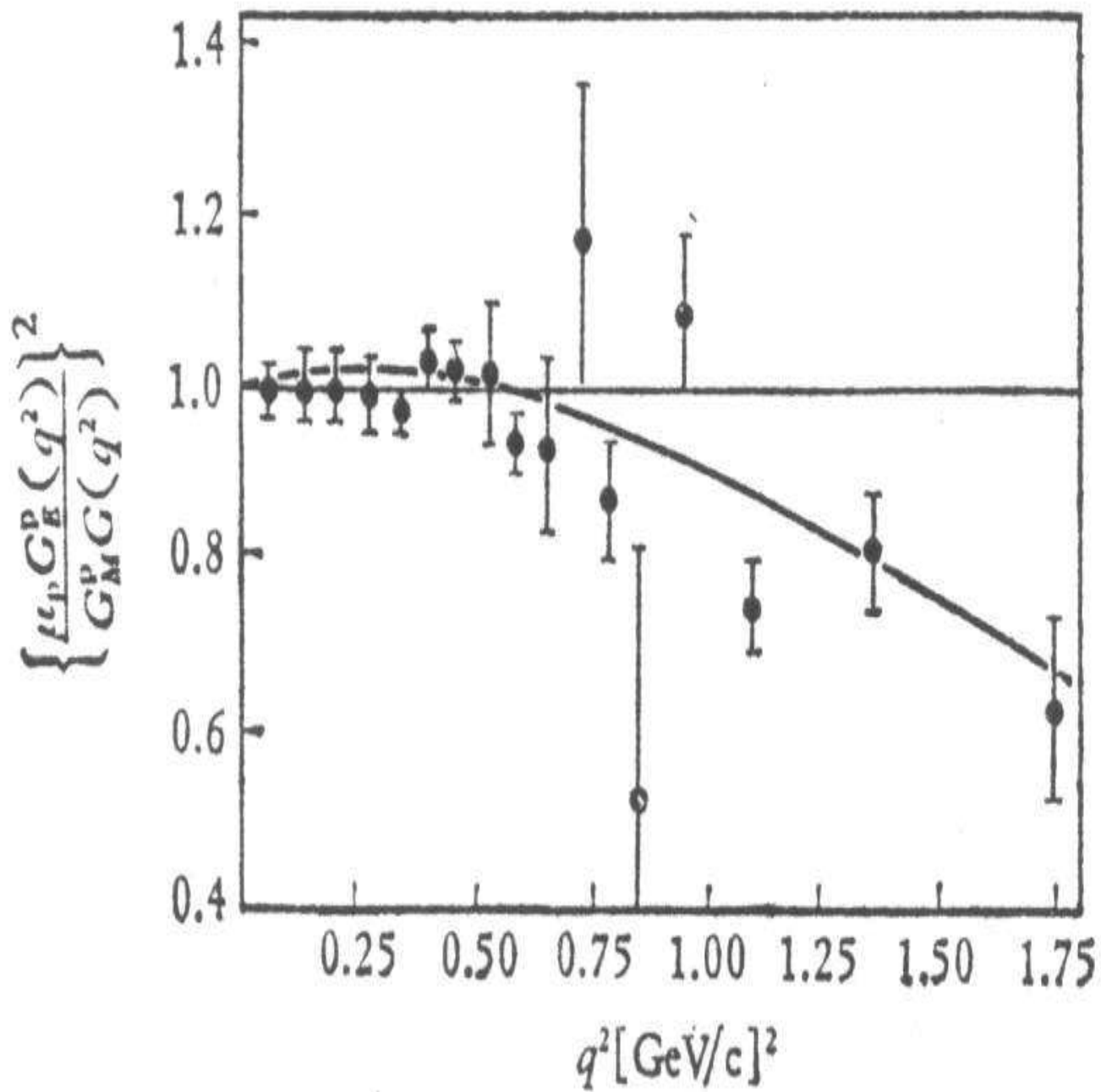


FIG. 1.

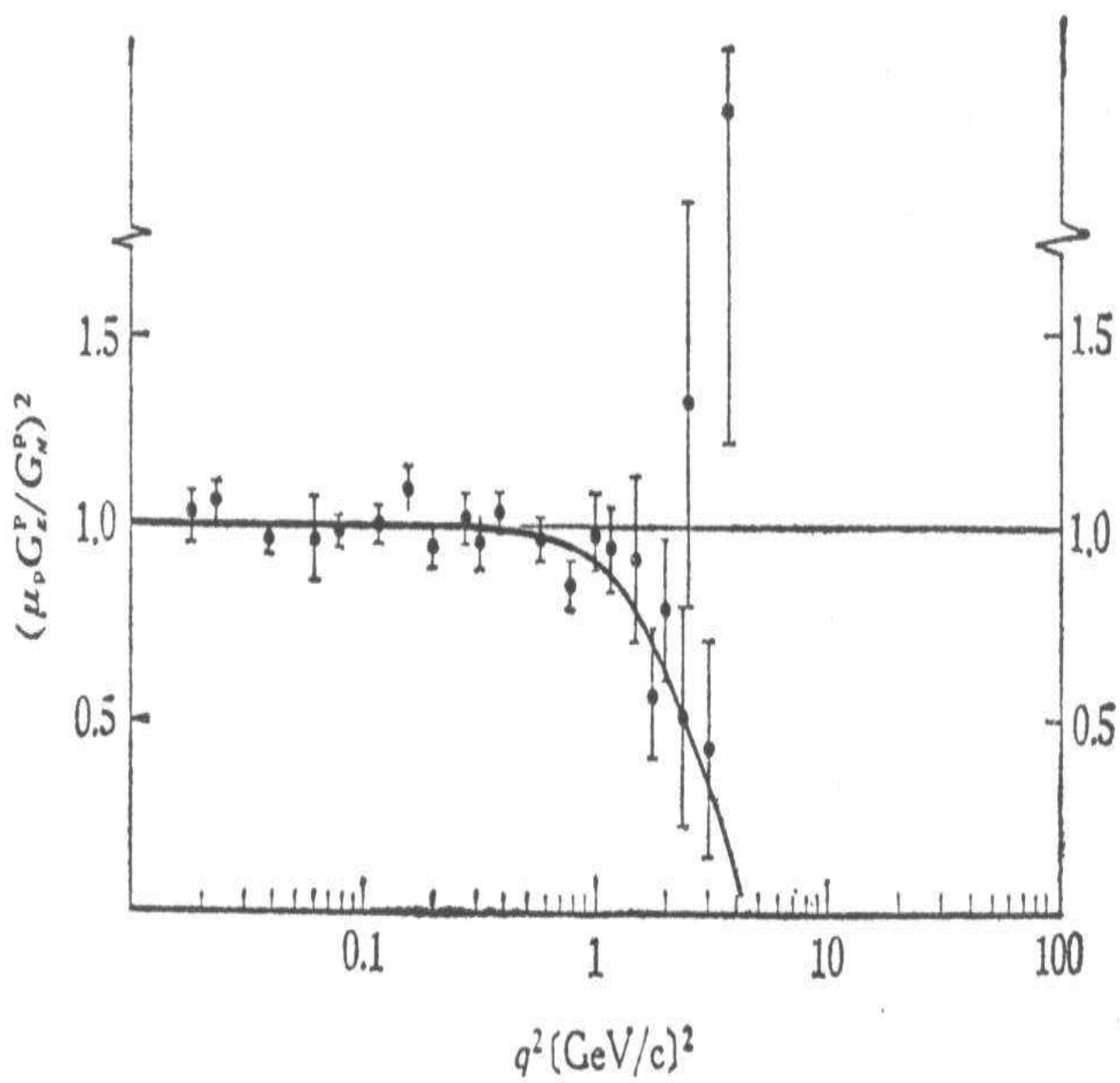


FIG. 2.

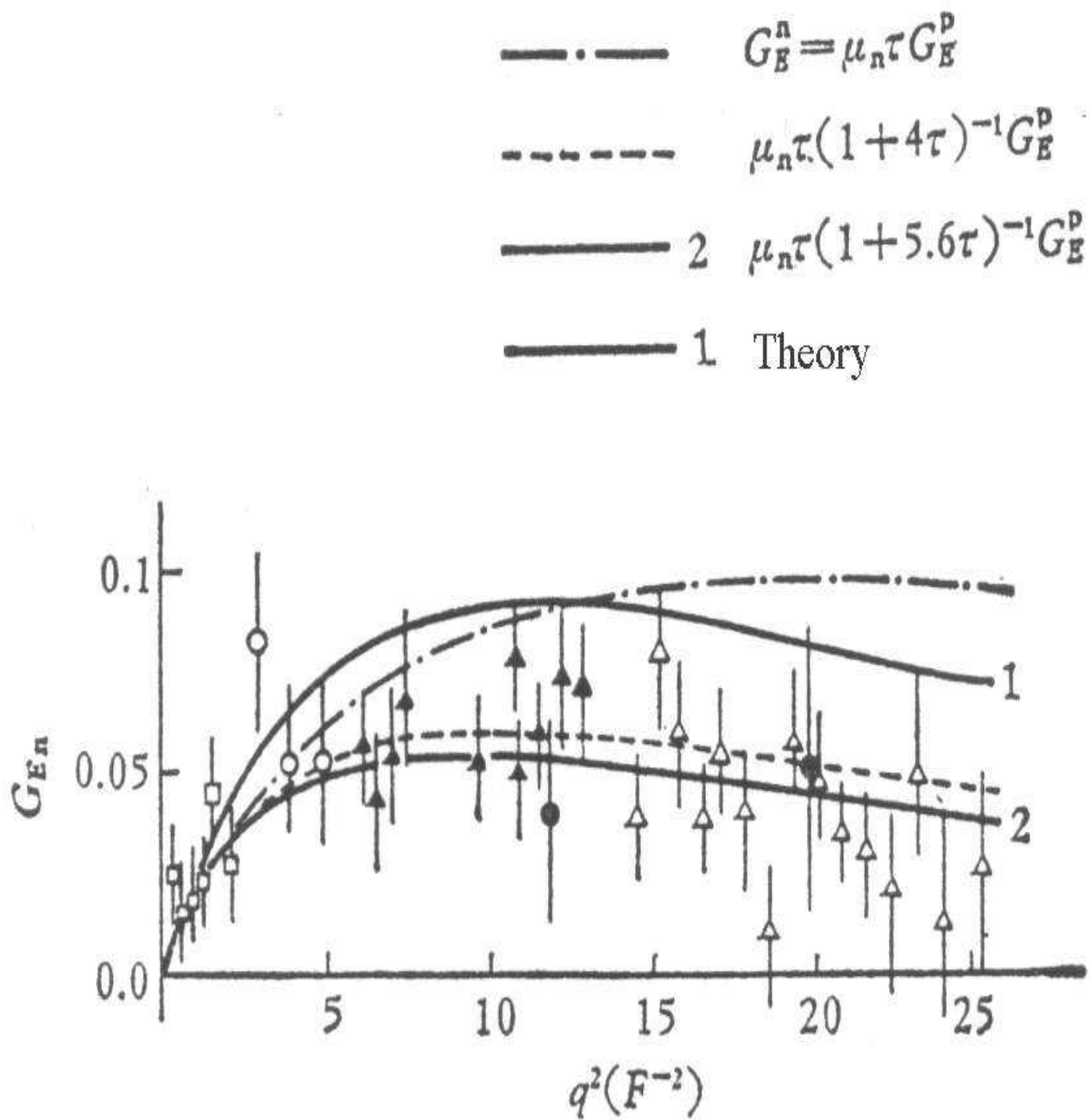


FIG. 3.

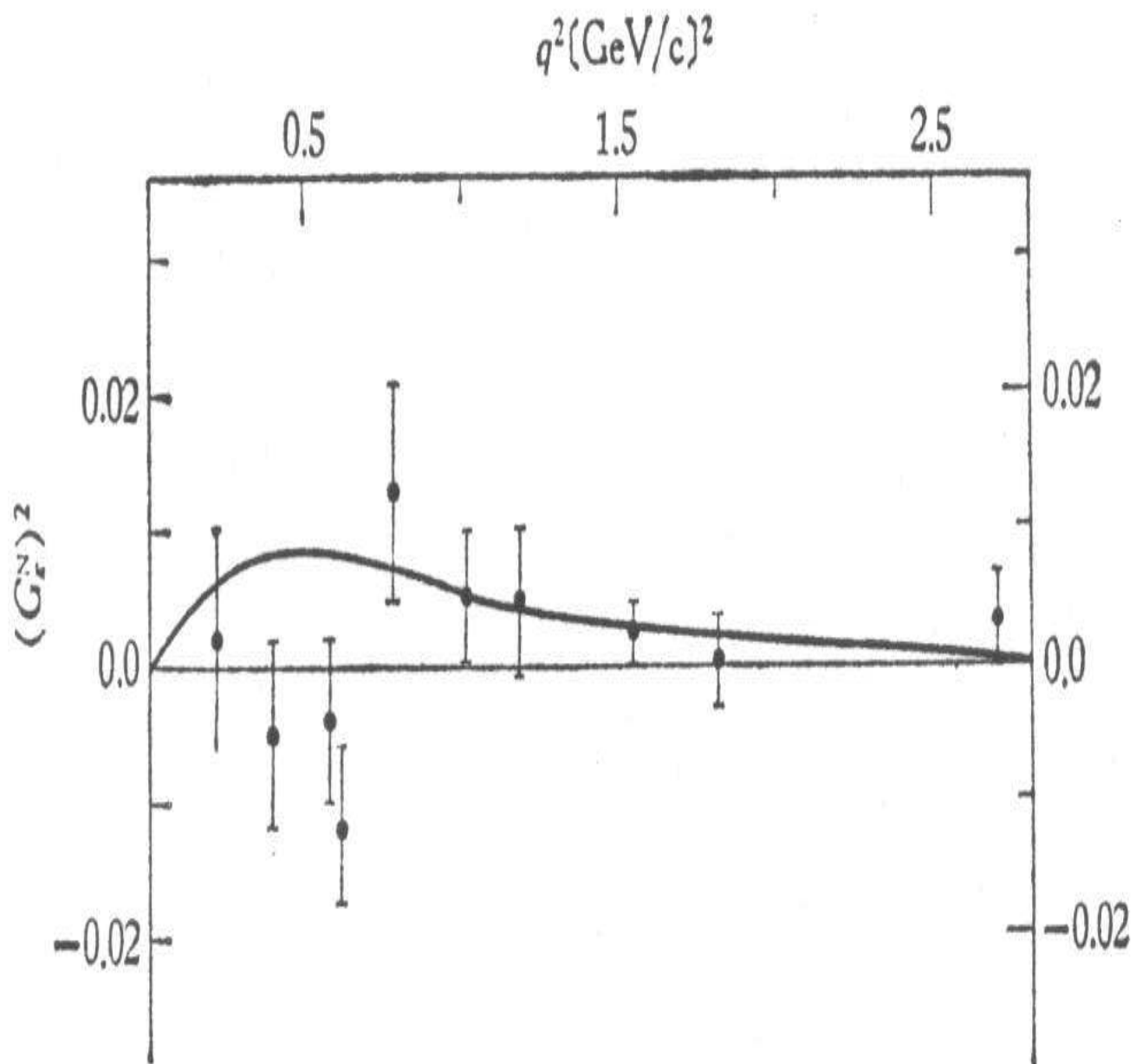


FIG. 4.

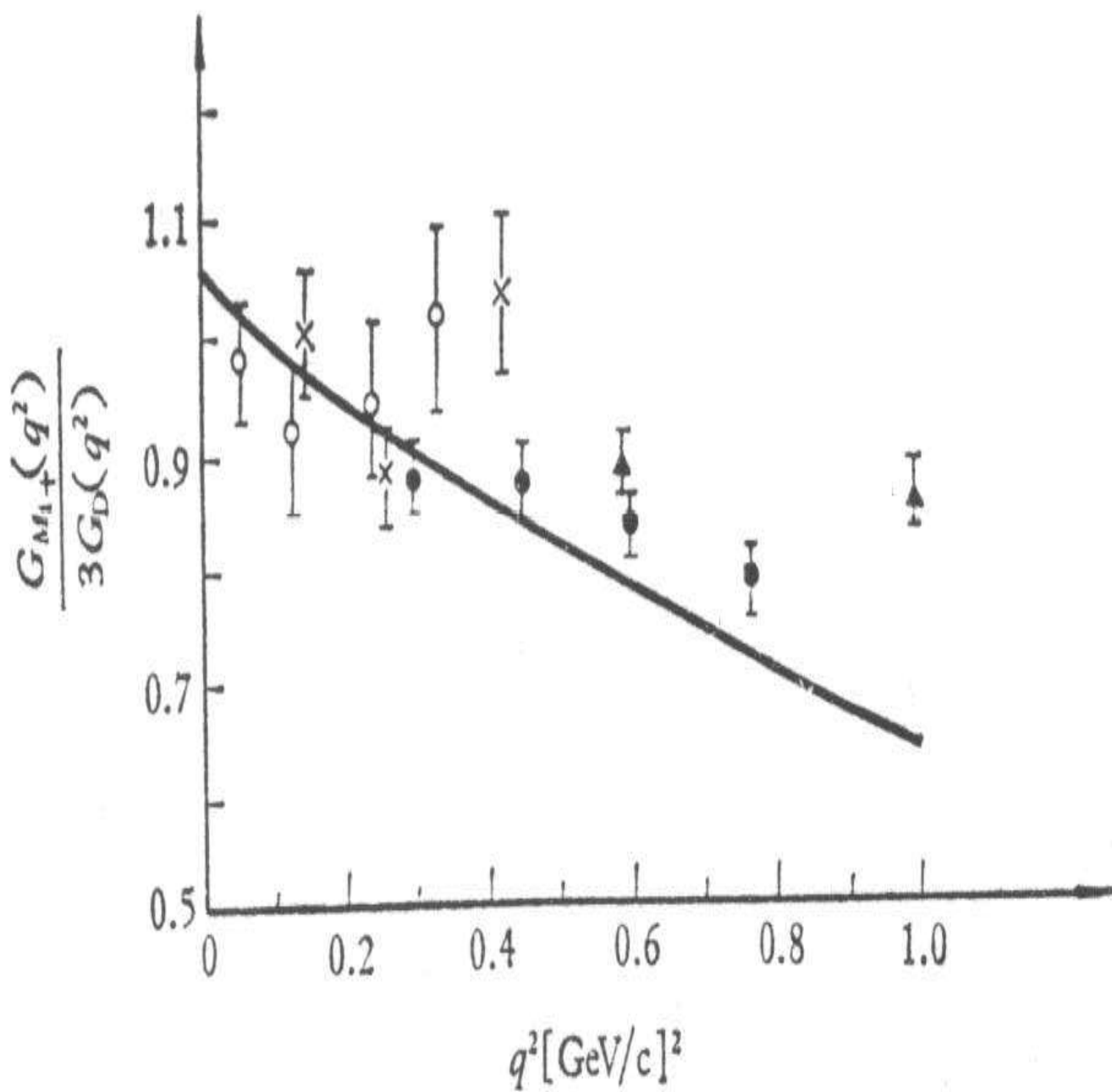


FIG. 5.

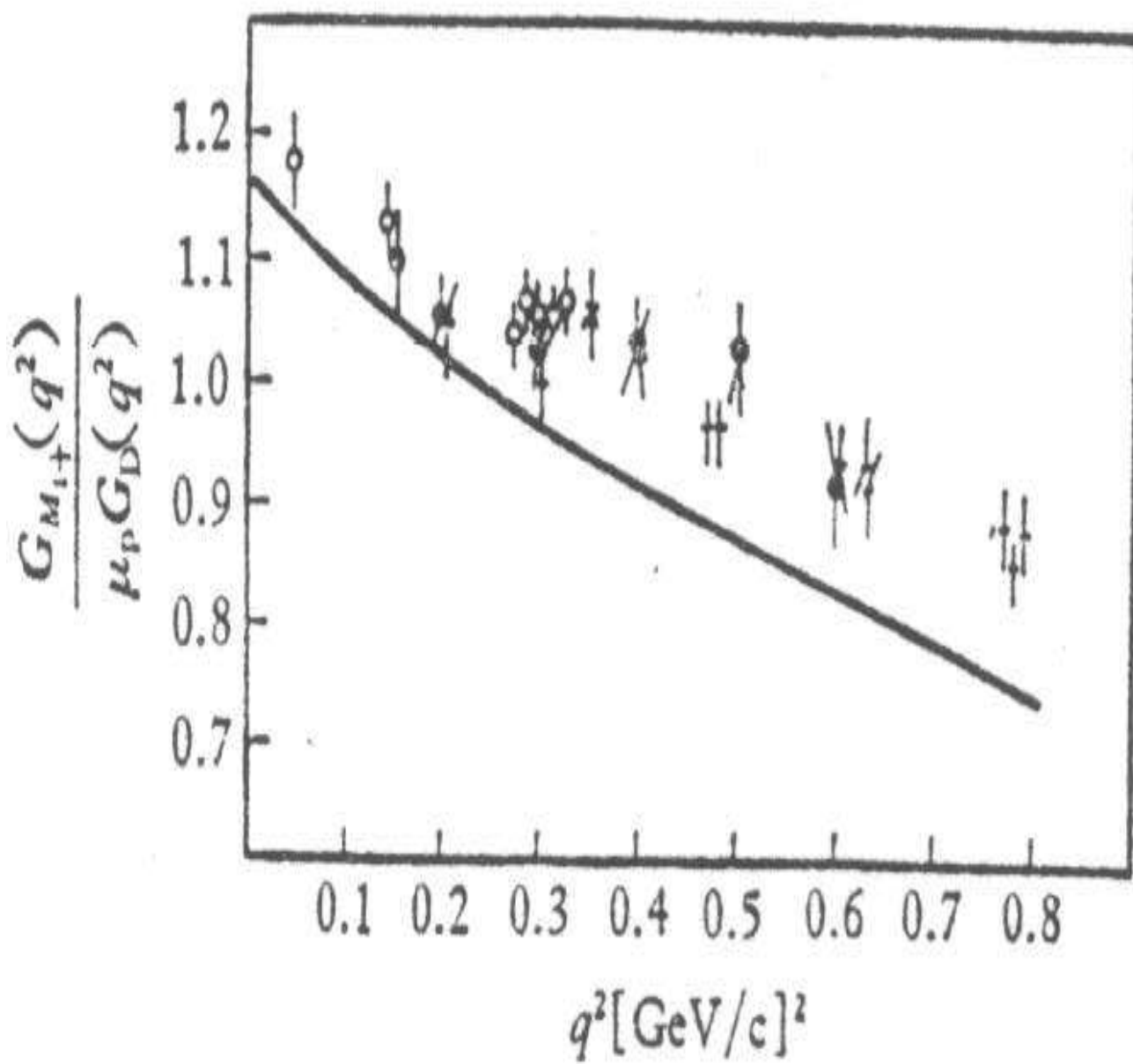


FIG. 6.

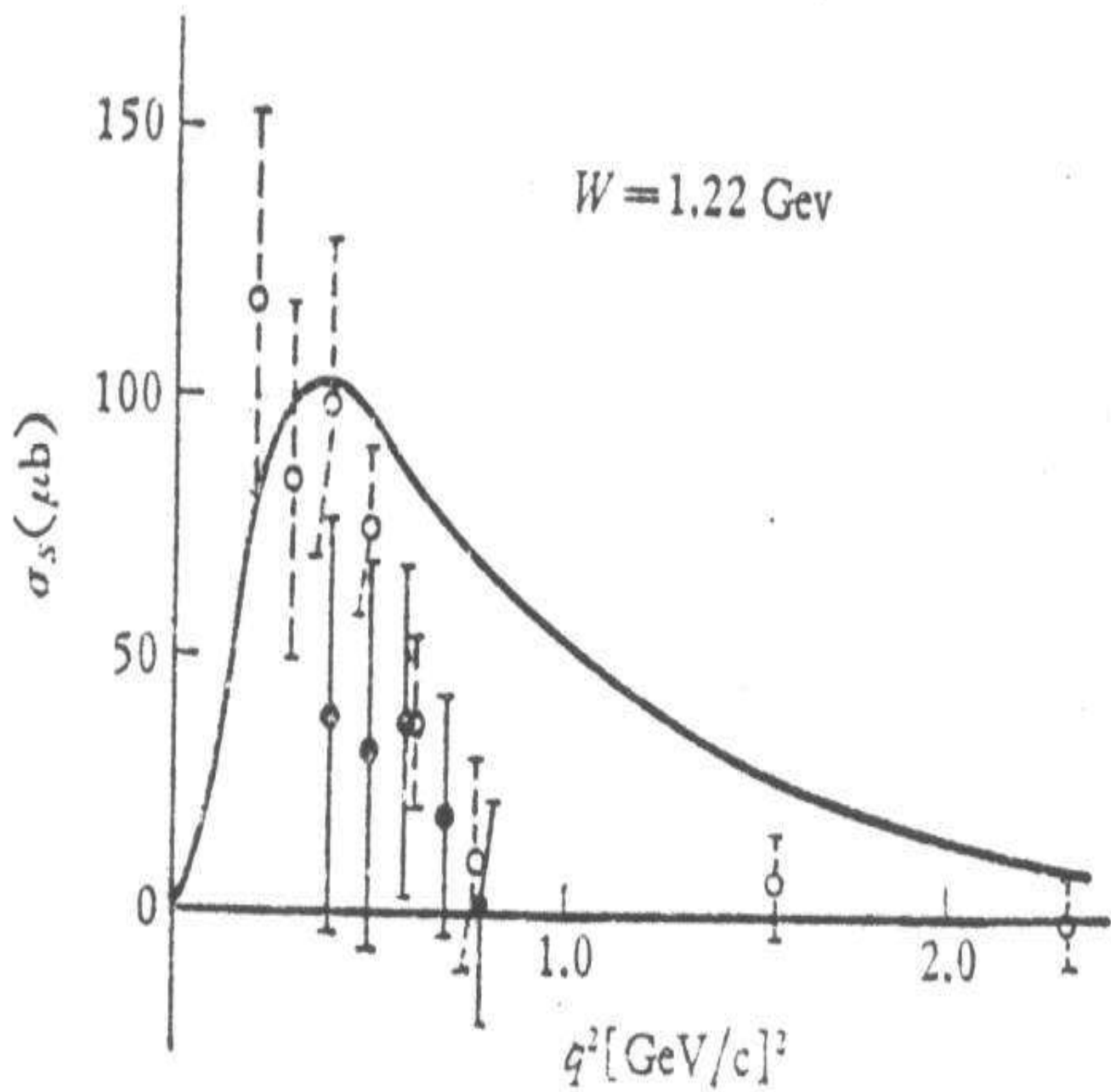


FIG. 7.